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IDEO-MOTOR ACTION[1]

THE problem of ideo-motor action has again been brought to the foreground by Professor Thorndike, who has called in question the orthodox view that an idea of a movement, in and of itself, has power to produce that movement. Pedagogical methods resulting from this doctrine, such as imitation, teaching children how to do given acts by acting the part before them, and reading tales of the noble and heroic as a means of developing character, are dismissed as factors in education. The reasons for this conclusion, I believe to be well grounded and to require no review. But they raise the questions: Are we, then, to conclude that ideas are of no use at all in conduct? If ideas of *movements* do not produce action, what sort of ideas *do* lead to action and *are* of value in education and pedagogy? It can not be denied that all our teaching, preaching, writing, and legislation are based upon faith in the motive power of ideas, and it is of primary importance to know what sort of ideas will lead to the results desired by these activities. The most positive statement upon the latter question that I find in the recent "Educational Psychology" is the following: "If the doctrine of this book is true, suggestion will succeed in so far as it is a process of manipulating a person's ideas and attitudes so as to get him into a situation to which the desired response rather than another is connected by the laws of instinct, exercise, and effect."[2] From this and other passages I infer that ideas have some efficacy in getting a person into the right situation, and function in that situation through some stimulus-response connection. At any rate I shall take this suggestion as the text of my paper and, upon this basis, attempt to show in some detail the value, functioning, and working of ideas.

The question of the value of an idea is as old as philosophy, but, strange to say, twenty-seven hundred years of speculation have failed to bring any agreement upon the solution. Whether this is a refutation of the speculative method or a proof of the insolubility of the problem I shall not try to decide. But it appears that an

[1] Read before the Psychological Club, Columbia University, March 12, 1914.

[2] Thorndike's "Educational Psychology," Vol. 1, page 293.

ancient statesman and a modern philosopher do agree upon the function of thought, and both upon empirical grounds. These are Pericles and Dewey. The former attributed the power of Athens to the power of thought. He said: "The great impediment to action is, in our opinion, not discussion, but want of that knowledge which is gained by discussion preparatory to action. For we have a peculiar power of thinking before we act, and of acting, too; whereas other men are courageous from ignorance, but hesitate upon reflection." The latter, viewing thought from a biological standpoint, came to the conclusion that its business is not to transcend nature or to create in idea a mystical and beautiful object of contemplation in the form of an eternal and absolute world, but to direct conduct and solve its problems. Such an agreement between a practical man and a philosopher is as worthy of attention as it is rare. When a philosopher, after reflection upon the power of thought, decides not to set it up as a divinity in some far away world of magic and wonder, but says that the place of its birth is not too humble for its functioning, it is but a step to the suggestion that thought, like other natural functions, is subject to experimental treatment, and that possibly this method as the opposite of the speculative will tell us more exactly how thought controls and directs conduct.

In agreement with this suggestion we shall view ideas or thought as a form of conduct and subject to the principles controlling conduct in general. To discover, then, the action of thought we shall find it helpful to consider genetically the conditions under which conduct occurs or responds.

Beginning with the behavior of the lowest animals, we notice that conduct is always in response to some stimulus coming from the environment, and further that the character of the stimulus determines the character of the response. For example, if an amœba is given a strong mechanical stimulus, it moves away; if it is given a weak mechanical stimulus, it moves toward the stimulus; and if a small particle of food is placed at its side, it envelops it.[3] If the anterior end of an earth-worm is touched, it creeps backward; if the side is touched, it turns away; if the posterior end is touched, it creeps forward;[4] and if it is thrown on its back, it twists around upon its ventral side.[5] The "flexion," "scratching," and "clasping" reflexes in a frog are other examples.

If we pass to higher forms, the stimulus-response principle holds throughout; but a great variety of objects may act as a stimulus setting off an equally great number of responses. The connection

[3] Jennings, "Behavior of Lower Organisms."
[4] Jennings, *J. of Ex. Zool.*, Vol. 3, page 435.
[5] Pearl, *Quar. J. Micro. Sci.*, Vol. 46, page 509.

between particular response and particular stimulus is, however, definite and is determined by the character of the situation on the one hand and the physiological condition on the other. For example, the perception of a wire loop, a button, a string, a thumb latch, a door, etc., by a hungry dog or cat placed inside a box having food on the outside may set off the necessary movements for getting out and securing the food. If a hungry South American cebus monkey[6] is placed in a cage and an experimenter with some food is on the outside, the monkey can learn to reach for food when the experimenter picks it up with his left hand, or shows it a white, diamond-shaped card-board, or a single brown card, or a card with a black ring on it; or if he picks up food with his left hand from a pile directly in front of the bar, or picks it up from a small brown pasteboard box. The monkey learns not to reach for food or change his reaction from the ordinary when the experimenter picks up food with his right hand, or shows it a black, diamond-shaped card-board, or a white and gold card, or a card with two black concentric rings on it, or takes food from a dish a short distance in front of the cage, or when he swings his left forearm diagonally around from his right side, or picks up food from a white crock, or a small saucer. The monkey was taught these various discriminations by being rewarded with food when he made the reaction desired by the experimenter, and not rewarded when its reactions were otherwise.[7]

If rewarded with food after a successful trial, a trained hungry rat will run a complicated maze without error as soon as it is placed inside it. If placed before a box containing food and banked with sawdust, or if the door is pasted shut with strips of paper, the perception of the sawdust or the strips of paper will, after a learning period, lead to the proper reactions for getting to the food.[8] Raccoons can learn to climb upon a box to be fed when a high note is sounded or when a large card is shown, and to stay down when a low note is sounded or when a small card is shown.[9] Further examples need not be cited to show that animal learning is dependent upon the perception of particular stimuli under particular conditions and situations.

We shall now pass to human learning. Here experimentation shows no exception to the stimulus-response principle. Apparently there are no reactions, however refined, which do not take their origin from a sensory cue of some kind. Just as there is no spontaneous generation of fermentation or life so there appears to be no spon-

[6] Thorndike, *Psych. Rev. Mon. Sup.*, Vol. 2, No. 4.
[7] Thorndike, *Psych. Rev. Mon. Sup.*, Vol. 3, No. 5.
[8] Small, *Am. J. of Psych.*, Vol. 12, page 206.
[9] Cole, *J. of Comp. Neur. and Psych.*, Vol. 17, page 211.

taneous behavior of the nervous system involving only the so-called motor paths. On the contrary, control of voluntary movement depends upon perception of the adequate stimulus. One of the first experiments relating to this point was made by Bowditch and Southard, who tried to touch a given point first by fixating it with the eyes, then closing the eyes and touching the same point by relying upon the muscular sense. They found that the movements were more accurate when accompanied by vision.[10] Bair trained fourteen subjects to move their ears with the *retranens aurem* muscle, usually an entirely unused muscle. He found that they succeeded after they had isolated and could attend to the particular sensation resulting from the movement of this muscle.[11] Professor Woodworth made an experiment in which he aimed to move his big toe while keeping the others quiet, and found that he succeeded after he could once exclusively attend to the sensation resulting from its movement.[12] He concludes that the sensation of movement is necessary in the acquisition of voluntary control; that a reproduced image of the sensations is valuable in reporting whether the result has been attained; but that the power to recall an image at will is not equivalent to acquiring voluntary control; and that the latter does not depend on kinesthetic images. He later made a more extended study of the same problem, in which he discovered that images are unimportant for the control of simple automatic acts like opening the eyes or wagging the jaw. Such acts can be performed immediately upon the reception of the external stimulus without the intervention of imagery. The fact that images are unimportant in such cases does, however, not prove that they are equally so in the learning of complex voluntary acts like typewriting or telegraphy. Another important contribution in this study is a pointing out of the value of "set" in voluntary control. Upon this Professor Woodworth says: "When a man confronted by a novel situation observes this and that feature of it in turn, each new perception leaves behind in the nervous system a temporary adjustment to the feature observed, until the whole situation becomes, not clearly mirrored in any one moment of his consciousness, but dynamically represented by the sum or resultant of these partial adjustments. If he then thinks of some change that he can make in the situation and decides to make it, the definiteness of his intention is not contained wholly in the field of attention at that moment, but depends upon the total neural set, and so on the total situation."[13] Later investigators, as we shall see, have con-

[10] *J. of Physiology*, 1881, pages 3, 232 ff.
[11] *Psych. Rev.*, Vol. 8, 1901, pages 474–510.
[12] "Le Mouvement," pages 330 ff.
[13] "Garman Memorial Volume," page 391.

firmed this conception, but have found that a long practise series is required to make the organization of "set" definite and effective.

A very important study on voluntary control has been made by Miss Downey,[14] who experimented with handwriting for the purpose of determining the importance of imagery. She found that while her subjects were dependent on sensory cues of some kind for the control of the writing process, no two used the same imagery throughout. Three of her subjects depended primarily upon automatic or muscular control; and four, on conscious control. But all depended on visual cues to some extent, it being most prominent in those using conscious control. Auditory imagery was prominent in three, and conscious grapho-motor in two. These results may be illustrated by referring to some of her tests. When her subjects wrote blindfolded, all showed mistakes in spacing, alignment, and in beginning the lines at the margin. Blindfolded writing with the left hand showed further deterioration in these respects. In inverted writing one succeeded by vocalizing the words and starting with a definite intention to invert. Two started with visual cues, but afterwards depended on muscular control. One visualized the letters upright and then inverted them mentally and wrote. One visualized the letters upright, inverted them, reversed, projected the image on the paper, and then wrote. Two subjects failed in this process because of inadequate sensory cues. For example, D., who was dependent on visual imagery, could not adequately visualize the letters upside down. These individual differences are further illustrated in the tests in which the subjects were required to write with a tablet placed on the forehead, or on the top of the head, or parallel to and at right angles with the back, or over either right or left shoulder. Three succeeded in these various forms by imagining the results as in normal writing such as looking down on the writing from above and at the left. Three others had no visual imagery, but depended upon kinesthetic reports and anticipatory motor cues. The peculiar dependence of the result upon the character of the imagery is illustrated in the case of S., who, in writing from left to right on top of the head, saw the letters as from below, with the result that the writing was reversed. But when he saw them as from above the writing became normal. There was a similar result in the case of H., who wrote mirror fashion upon the forehead, but at times reproduced normally when she imagined herself seeing the letters from the front. The best tests, however, which Miss Downey made to bring out the importance of imagery were such as writing with eyes open while counting aloud, reading aloud, counting mentally the number of times a particular word occurred in a rhyming list read

[14] *Psych. Rev. Mon. Suppl.*, Vol. 9, 1908, pages 1–148.

aloud by the experimenter, or writing blindfolded and counting aloud. From these tests we may expect that if a writer depends on a given sensory cue, a distraction affecting that cue will raise his writing time more than any other distraction. For example, if a writer depends upon articulatory or vocal motor control, reading aloud will raise his writing time more than writing blindfolded. The results were in agreement with these expectations. If a writer's usual sensory cue was interfered with, his writing time was raised much above the normal, but this was not the case when the distraction affected an unusual cue. When those who depended on voluntarily produced imagery had their attention completely distracted, they found it impossible to write. Another interesting result was that when a used cue was distracted, the writer could shift to another, *e. g.*, change from visual control to articulatory or auditory. But the principal result of the experiment is that the control of every voluntary act depended upon some sensory cue. If this cue could not be secured in perception, it was supplied by imagery.

Judd[15] assisted by McAllister, Steele, Cameron, and Courten made a series of experiments that have a bearing upon our problem. These included three experiments upon illusions, one upon reaction time, and one upon practise without knowledge of results. In the experiments upon illusions the eye movements were photographed both at the beginning of a practise series, when the illusion appeared, and at the end, when the illusion disappeared. In the Müller-Lyer illusion it was found that before practise the eyes made many more pauses in the underestimated area than in the overestimated area; but after the practise series, the eyes moved with equal facility along the entire line and made an equal number of pauses in each field. In the Poggendorf illusion it was found that before practise the eyes moved along the oblique irregularly, making frequent and long pauses at the points of intersection with the verticals. After practise the eyes moved along the oblique line with almost no deflection, with fewer and more equally distributed pauses, and with shorter and more rapid motion. In the Zöllner illusion it was found that before practise the eyes moved along the long parallels very irregularly, making many pauses at points of intersection, many deflections, and many twists. After practise there was a close adherence to an undeflected and regular movement along the long lines with pauses at regular intervals. In the reaction experiment the movements of the reacting hand both before and after the reception of the get-ready stimulus were recorded on waxed paper. The record showed a marked unsteadiness of the hand after this signal, indicating an increasing tendency to react. In the practise

[15] *Psych. Rev. Mon. Suppl.*, Vol. VII., 1905, pages 1 ff.

experiment without knowledge of results the subject was required to indicate the projection of a series of oblique lines both above and below a given horizontal. Two hundred trials upon each line distributed over a period of ten days showed no improvement. A series of tests with knowledge of results was then made upon two obliques, one forming an angle of 60 degrees above the horizontal, the other an angle of 45 degrees below the horizontal. A practise series distributed over ten days corrected the error with the first line, but increased it with the second. The correction of the first line was effected by placing the projection at a point higher than the first point indicated. The same habit was transferred to the second line with adverse results. What is significant in the illusion experiments for our purposes is that when the perception of the illusion disappeared the sensory stimulus was different from what it was in the illusion. The reaction experiment indicated that the reception of a sensory stimulus tends to be followed by a motor response even when it should be inhibited, and the reaction experiment indicated that perception of one's results of reactions to be perfected changes the character of those reactions. In each case control of the reaction depended upon getting the right sensory cue.

Another series of experiments bearing directly upon our problem are the so-called practise experiments. Hitherto attention has been directed principally to the amount of improvement of a given function, its persistence, and the form of the practise curve. A careful review of the literature, however, points to the conclusion that improvement in a given function depends in a great measure upon the forms of perception to which the desired movements are made. Leuba and Hyde,[16] who had students practise writing English prose in German script by the aid of a copy of the German script alphabet, found three stages of progress. In the first stage the student pronounced in inner speech the English letters separately, then associated them with the perception of the German letters which were pronounced in inner speech and then written. In the second stage, the inner speech process dropped out and the writing of the German letters followed immediately upon the perception of the English letters. In the third stage the English letters were no more perceived separately, but in word and phrase groups and written in the same units. It is interesting to note that in each stage the units of reaction correspond with the units of perception.

Rowe[17] in an experiment designed to analyze the sensory processes in voluntary control had four subjects practise, until the reactions became automatic, writing ten standard words by pressing

[16] *Psych. Rev.*, Vol. 12, 1905, page 357.

[17] *Am. J. of Psych.*, Vol. 21, 1910, pages 513 ff.

small rubber bulbs arranged so as to correspond in number and letter order to the lower row of keys in a Blickensderfer typewriter. The letters were indicated by means of a cardboard placed in front of the bulbs. In general, during the first stage of development, the subjects had to look at the stimulus words, pronounce them in inner speech, pronounce each of the letters in inner speech, and find them on the cardboard, and then look at each letter carefully while pressing the proper bulb, often dividing the attention between the cardboard letter and the pressing movement. In a second stage, the perception of the cardboard and the keyboard were eliminated and the reactions followed upon the memory image of the copy. In the third stage, after the ten words could be written in one series of coordinated finger movements, perception of details was eliminated and there remained only a general "set" characterized by attention to the situation as a whole in which were present general bodily, tactual-kinesthetic, and visual sensations. Inner speech still occurred in some cases, but was unimportant, for many of the reactions preceded the inner speech. His conclusion from his observations is that without sense-processes relevant to the situation purely ideational processes are incompetent to initiate movements. Sensations are the *sine qua non* of consciously controlled acts. In the development of voluntary control the correct movement is first made reflexly, certain sense elements connected with it are then caught by the attention and are made functional through its effect. In the highest volitional processes both clear perception and clear imagery are necessary, but attention may leave the process through the substitution of the general symbol for the details of the original.

Another experiment which shows the close dependence of voluntary control upon the form of perception is Book's study of skill in typewriting.[18] In this connection the short-circuiting in the perceptual processes in the development of speed is interesting. This may be illustrated with reference to those using the touch method. The subjects committed the keyboard to memory, after which it was screened and the process of writing begun. The subjects first got the copy; second, pronounced the letter; third, located it mentally on the keyboard, for example, from a visual image of its correct position; fourth, got the finger on the proper key by first locating its row and then counting and feeling the individual keys until the proper one was found; fifth, pronounced the letters again and made the correct movement. It is little wonder that during the early stages the letter to be written was sometimes forgotten during this complicated process. Improvement was made by a gradual fusion of these steps. The initial spelling fused into the sight of the letter

[18] "Psychology of Skill," 1908.

so that the remaining processes followed successfully upon sight of the letter. The mental locating of the keys and the complicated finger movements made to find the right key fused into a motor-touch image so that as soon as the letter was seen it could be correctly located by getting a "feel" of the movement necessary to hit the desired key. Later the motor part of the image dropped out and the correctness of the movement was recognized by touch alone. Finally the inner spelling also dropped out so that the sight of the copy led directly to the proper reaction. The inner spelling, however, reappeared whenever difficult words were met.

In a second stage the learner reads and writes in syllable and word units. One subject describes this process as follows: "An easy word is taken in as a whole; a few short easy ones are taken in as a connected series. A long and difficult word may be broken up into groups of easy familiar combinations, or in case of extreme difficulty, it may be incipiently spelled letter by letter, and by attending to each individual movement. This way of spelling words or taking in familiar parts of them, and the manner of the movements seem to run closely parallel all the time. I take a thing in on the keyboard just as I take it in in the copy—by letter, by combinations of letters, by words or groups of short easy words."[19] The same subject is again quoted as follows: "A word simply means a group of movements which I tend to as a whole. I seem to get beforehand a sort of "feel" of the whole group which is run through with that sort of conception and direction of attention."[20] In learning to make group movements inner spelling was at first necessary to give the right sequence, but when the groups became familiar it faded away.

In the expert stage, phrases, clauses, and sentences became the unit of attention. Movements were no longer attended to in word groups, but attention was directed toward getting through with the sentence as a whole and with performing the movements in the right succession. The eyes were kept continuously on the copy a number of words ahead of the hand. The pronouncing of the words served to start and control the series of movements.

Such in outline is the order of development in learning to typewrite by the touch method. The genesis in the sight method is not characteristically different in principle. It will be found that the genesis is of the same character as other investigators have found, beginning with sharp attention and perception of every detail and ending with attention and perception directed to the process as a whole. It may be asked why this cumbersome process must be gone through in the learning of skill and why we can not begin writing in sentence units

[19] *Idem*, page 38.

[20] *Idem*, page 39.

at once instead of worrying with the drudgery of letters. One reason may be the physical resistance of the muscular tissue to a sudden adaptation to such complicated movements. But I doubt whether this is the chief difficulty, because the muscles of the young are extremely flexible. I venture to suggest that the difficulty lies rather in the nervous system, in its inability largely to get adequate sensory stimuli. A sentence can not be written upon the perception of a single letter. This requires a perception of the sentence as a whole and a feeling of the entire series of movements as a whole. In other words, the unit of reaction must correspond to the unit of perception. The converse of this proposition may be equally true. It is altogether likely that the limit of the unit of perception is determined by the limit of the reaction. But this does not gainsay the contention that voluntary control depends upon perception.

If it is true that voluntary control depends upon the perception of the adequate stimulus, then we should expect a loss of this function to follow upon an elimination of the sensory cue. There is some physiological, pathological, and clinical evidence in favor of this view, but not as much as we could wish for. Bernard[21] made an experiment in which he produced total anesthesia of a frog's hind feet by cutting the sensory spinal roots innervating those parts. The result was a failure in the coordination of the feet. Mott and Sherrington made a similar experiment on a monkey by cutting on one side of the cord the dorsal roots of the nerves supplying the arm and the leg. The authors conclude that volition in these members was absolutely abolished by this operation.[22] Sherrington, reporting on a second experiment upon this problem, concludes that destruction of sensitivity in particular regions causes objectively observable disturbances of movement. "The effects of the anesthesia upon the musculature of the part are three—(1) paralysis, (2) ataxia, (3) atonia."[23] Munk in repeating the experiment of Mott and Sherrington concluded that voluntary control does not depend upon the muscular sense alone, but may be recovered by practise through other senses like vision. Flourens noticed that cutting the semicircular canals in pigeons caused confused and disordered movements.[24] Ewald, repeating the experiments, found that extirpating a canal on one side caused a twisting of the head, an unequal extension of the feet, and an inclination of the body toward the injured side.[25]

[21] "Leçons sur la Physiologie du System Nerveux," Paris, 1858, pages 1, 251.

[22] *Proc. Roy. Soc.*, Vol. 57, pages 481 ff.

[23] *Ibid.*, Vol. 61.

[24] "Researches Experimentales sur les Proprietes et les Functions du System Nerveux," 2d ed., 1842, pages 452 ff.

[25] "Physiologische Untersuchungen über das Endorgan des Nervus Octavus," 1892.

There was a loss of normal balancing movements on that side. Turning to some pathological and clinical evidence, Gley and Marillier[26] report a case of a patient deprived of all sensibility in the upper half of his body. When blindfolded the patient's arms could be moved in any direction by the experimenter without his being able to describe their position, he could not discriminate between a weight of 250 grams and one of 1,850 grams, nor tell the difference between hard wood and soft rubber, nor make movements upon command. Bleuler[27] describes a case of total anesthesia on the right side, in which the patient could not move his right arm except when he saw it. A. Pick[28] reports a case of complete loss of deep and superficial sensibility on the right side. The patient could open and close her right hand when she fixated upon it, but not when the hand was outside the center of vision. Berkeley[29] reports a case in which there was total loss of thermic, pain, olfactory, gustatory, equilibrium, pressure, and weight sensations, an almost total loss of vision, a partial loss of muscular sense, and a progressive dulling of hearing. The patient, who had not lost sanity, could not move the indicator in a dynamometer upon command, although she made every effort to do so. She could not help herself in any way.

Hoppe[30] describes a case in which the muscular sense and the sense of the position of the limb were almost absent. The right arm seemed completely paralyzed and the patient could not move it upon command. But she learned to do so by watching the doctor's hand move and then repeating the movements.

These cases point to a close dependence of voluntary control upon some kind of sensation and that, in case of need, there is possibility of transferring control from one sense to another. Such shiftings are similar to those reported by Miss Downey.

What we have gained from this review of experimental evidence on the sensation-movement relationship is: (1) that both human and animal behavior takes place in response to sensory stimuli; (2) that the character of the stimulus determines the character of the response; and (3) that control of the response depends upon control of the stimulus. Propositions (2) and (3) are still open to further experimentation. But the findings of Bryan and Harter, Book, Leuba, and Rowe upon the specific correspondence between perception units and reaction units, and Judd's discovery of the disappearance of illusions after the sensory stimulus was under control, leave little room for doubt. At least it is safe to accept them as

[26] *Revue Philosophique*, 1887, XXIII., pages 441 ff.

[27] *Archiv für Psychiatrie und Nervenkrankheiten*, 1893, Vol. 25.

[28] *Zeit. für Psy. und Phys. der Sinnes.*, Vol. 4, page 175.

[29] *Brain*, Vol. 23, page 14.

[30] *J. of Mental and Nervous Diseases*, Vol. 32, page 145.

favorable hypotheses. It will be observed that no fiats of will are reported, no so-called feelings of innervation, nor feelings of the muscles and joints in movement. These are superfluous intermediaries between the stimulus and the response. Control is obtained by attention to the stimuli and to the results of the movement. Perception of the results of one's movements is rather a stimulus for the following movement.

If the above conclusions are accepted we have not far to go to find the function of ideas. It will be recalled that Bair's subjects could move their ears after they could attend to the sensation resulting from their movement, and Professor Woodworth could move his great toe after he could attend to the sensation produced by its movement. In trying Bair's experiment upon my left ear, I discovered that after the ear once got into motion the new position of the muscles produced a new sensation. It was not a sensation of the muscle in motion, but a sensation of the muscle after this motion was completed. Attending to this sensation independently gave me control over the desired movement. This and the other experiment reported suggests that the function of ideas or cognition is to analyze the sensory stimuli to which conduct responds. These stimuli may then be caught by the attention and responded to voluntarily. A change in conduct is then effected through attention to a different stimulus. Why I respond to one stimulus rather than to another is due to a complication of factors such as those of instinct, habit, desire, and satisfaction, but in particular, to the interest of the present moment. For example, just now I am interested in writing this paper. I attend only to those things which have a relevancy to my problem. I see hundreds of books before me, but I open and read only a few of them. If I should read a classical periodical which I now see it would pain me as a waste of time. That is, I am adjusted to respond only to matters concerning the ideo-motor problem, and have developed a temporary set to that effect. I developed this set in response to a problem which was again brought vividly to my attention by reading Thorndike's "Psychology" because of a general interest in the science. I might continue giving account for my interest in psychology, and so on indefinitely; but the point I wish to make clear is that "set" is a temporary response of the nervous system as a whole to a present situation and is in turn determined by my past conduct. It is therefore due in part to present stimuli, but probably more to responses to stimuli in the past.

Reverting to the problem of the function of ideas, I wish to describe more in detail the process of analysis. For example, when a boy goes through the Columbia library and begins turning off the lights, the sensation produced is a stimulus to get out and to go home

for rest. Going out of the building, down wooden steps, and out at a narrow wooden gate is a stimulus to turn north. Coming to a show window in which are displayed wines and candies is a stimulus to turn west. Going up a hill and seeing iron posts on each side of stone steps is a stimulus to turn in. Before I had analyzed the characteristic stimuli at each turning point, I could not find my way without error. I turned west at the wrong block or in at the wrong door. I then set to work and made a comparison of the marks at each corner where I should turn and picked out certain easily distinguishable marks which were peculiar to each and which I could take as a never failing stimulus to turn in a certain way. After I had done this I could find my way home without apparently seeing anything. This analysis and fixation of the proper stimuli is accomplished through my ideas and power of cognition.

In the most highly technical scientific process the function of ideas is nothing different. Experimentation is simply an accurate means of finding and analyzing the proper stimuli for the formation of judgment and conduct. For example, before the time of Galileo, the Aristotelians said that bodies fall in proportion to their weight, that each body seeks its natural place, and that the natural place of a stone was on the earth and of a feather in the air, etc. These are accurate descriptions of our sense-perceptions in daily life. Galileo noticed from general observation that the velocity of a falling body was constantly on the increase and conceived the hypothesis that the velocity is proportional to the distance descended through. Finding, as he thought, a logical contradiction in this theory, he made another according to which velocity is proportional to the time of descent. Finding no contradiction in this view, he proceeded experimentally to verify it, by rolling balls down an inclined plane and measuring the times of descent. The time was measured by means of a water clock consisting of a wide vessel with an orifice at the bottom which was closed with the finger. As soon as the ball began to roll Galileo removed his finger and let the water run into a balance. When the ball reached its terminus, he closed the orifice. He found that while the times or units of water increased simply the distances increased quadradically—*i. e.*, that the distance varies directly as the square of the time of the descent.[31] What prevented Aristotle from reaching this conclusion was that he had no means of analyzing his gross perceptions. Galileo devised a means, in the inclined plane, water clock, and balls, which enabled him to make accurate perceptions and find the proper stimuli upon which to form a judgment. This way of devising an experimental method of analysis is the function of ideas.

[31] Mach, "Science of Mechanics," page 130 ff.

In this connection I must also make a brief reference to the work of Pasteur[32] on fermentation. Before and during his day fermentation was believed to be due to spontaneous generation. Grape juice and milk soured very soon after extraction and apparently without the addition of new elements. The hypothesis of spontaneous generation was therefore an accurate description of the gross sense perception of the phenomena. Pasteur, however, submitted milk and other liquids to experimentation, using a more vigorous technique, and succeeded in showing that fermentation was due to foreign particles coming in contact with the liquid, particles in the surrounding air or in and around the containing vessels. His paper on lactic fermentation, which published this result, is a classic example of the power of an idea. A copy fell into the hands of a young surgeon of Scotland, Dr. Lister, who concluded from this paper that probably putrefaction in wounds was a form of fermentation caused by particles in the air and on the surgeon's hands and tools which came in contact with the wound. He therefore began operations in which the wounds and the surgeon's hands and tools were thoroughly cleansed by a solution of carbolic acid, and in two years discovered that out of forty patients upon whom amputations were made thirty-four recovered. The meaning of this may be gathered from the fact that during the same time in Paris where antiseptic surgery was not practised the mortality after amputation was 60 per cent., and during the siege of Paris almost every one operated on at the Grand Hotel, the temporary ambulance, died of purulent infection.

Another copy of this paper fell into the hands of one of Pasteur's countrymen, Davainne, who had observed small filiform bodies in the blood of animals that had died of anthrax,—a disease which annually killed 20,000,000 francs worth of domestic animals in France. Pasteur's paper suggested that probably the filiform bodies he had observed caused the disease in a manner similar to those causing fermentation. He verified his hypothesis by successfully inoculating healthy animals with the blood taken from the diseased animals. Koch, from Berlin, later took up the study and succeeded in isolating the particular germ causing the disease. Pasteur took up the study and by attenuating the virus containing the bacillus anthracis discovered a vaccine which rendered animals immune to the disease. This alone meant an annual saving of 20,000,000 francs to France, which gives a quantitative notion of the value of one of Pasteur's ideas.

We can not go into the study of Pasteur's work any farther.

[32] Vallery-Radot, "Life of Pasteur," Mrs. Devonshire's transl.; Pasteur. "Studies on Fermentation," Faulkner and Robbs's transl.

It is sufficient to mention that some of the results which sprang from the new ideas contained in Pasteur's study of lactic fermentation are antiseptic surgery, the cure of anthrax, hydrophobia, diphtheria, and in general the processes involving antiseptics and serum-therapy. The value of this resists a quantitative estimate. I refer to this not as a matter of history, but to illustrate the value and working of ideas. Pasteur by means of an exact technique succeeded in analyzing and isolating the proper stimuli with reference to fermentation. He puts his findings into language, and publishes them. The paper crosses the sea and falls into the hands of a sagacious reader in a foreign land. The result is antiseptic surgery. This alone illustrates the process. The reasons that, before Pasteur, surgery, diphtheria, rabies, etc., were the plagues and decimators of humanity are that no one knew to what stimuli to react in these phenomena. It was left to the ideas and cognitive powers of Pasteur to devise an exact technique and analyze those stimuli, and after that analysis, control at once followed and the dreaded evils vanished. The work of Pasteur in itself proves that ideas produce productive action and results when they are of the right sort and point out the right sort of stimuli.

If this paper has accomplished its aim in showing that the work and function of ideas is to analyze stimuli to which conduct responds, we are in position to understand why an idea of a movement does not produce it; and why imitation, the teaching of animals and children how to do certain acts by acting the part before them, or by putting them through the desired acts, and the reading of tales of the noble and heroic are usually ineffective in getting the desired results. They fail to point out the proper stimuli to which the desired acts are a response. On the other hand, such methods, as well as preaching, lecturing, making laws, and writing books, do yield productive results when they succeed in pointing out the proper stimuli for the conduct desired. Such activities need, therefore, not be stopped for fear that ideas will produce no movements, but it is necessary that they be directed toward the adequate stimuli.

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VALUES AND EXPERIENCE[1]

IN a recent paper[2] I tried to indicate how scientific thinking is to be recognized as only the more reflective stage in man's vital enterprise of exploring and manipulating the rich world about him,

[1] Read at the annual meeting of the American Philosophical Association, New Haven, December 29, 1913.

[2] *Philosophical Review*, Vol. XXII., pages 520–538.

a world primarily not of facts, but of meanings. From the same standpoint I should like to touch on some questions raised during the past year on the subject of values.

Forsaking the vast accumulation of petrified data piled up by the more intellectual operations of man for centuries, and throwing ourselves (shall I say, by an act of intuition?) into the heart of ordinary naïve human experience, we find ourselves in a world not so much of material persisting objects of brick and mortar as of weltering and stewing, promising and threatening, agents and forces. If nothing else, the world is one of change, or rather, of changes. Things are happening, coming and going, upward and downward, inward and outward, forward and backward. It is a dynamism of an incessant and unmistakable character.

It is not, however, a *mere* dynamism, but one that shows certain qualitative characters not connotated by that term. The agencies operative are agencies that throw themselves as it were into our attention. They stand over against us in a genuinely objective sense—threatening, appealing, coercing, attracting, repelling. They appear as good, ugly, bad, magnificent, wrong, beautiful, upright. In fact, they are *just* these: they are goods, uglies, bads, magnificents, wrongs, beautifuls, uprights. As such and only as such are they *there* at all. The original material of all human experience presents itself in this intimate and face-to-face manner. It is especially obvious in the more novel experiences of the adult and, we suppose, in the earliest experiences of the babe. The situation may be no further defined, but it has at least this romantic, this rich, this brimful character.

Moreover, if this be true of vague and novel moments, there is no warrant for maintaining that developed and intelligent moments are lacking in this character. The development and organization of the former into the latter involves no denial of the meaningful character—in fact, it requires the *presence* of it to furnish the very stimuli and clues to the development. This, now, is what we mean by the term "values." Generalizing, we may say that the world as experienced is a world of appreciative qualities, of *value* aspects. It is not an impersonal casing that compasses us about, but a multiplicity of guide-posts that may serve our human purposes and become linked with our personal fortunes.

A moment ago I called attention to the dynamic character of the world we live in. We saw it as an ever-changing flux. If, now, our interest in the value aspect of the world becomes a little reflective and we wish to look back into the experience to determine more accurately just in what this aspect consists, we shall find that our statement of the incessant alterations and variations must be modified.

A most significant feature of this flux is that it is not entirely haphazard, chaotic, without direction. Some changes seem to have worked themselves up into systems of changes, more and more coordinated, more and more organized, and form concretions within the whole dynamic flood, vortices appropriating to themselves more and more of the surrounding changes and rendering them less haphazard, more unified and correlated. Definite directions have thus emerged. And as these innumerable vortices, these concretions, strike out their numerous paths, the welter of changes about them tends to be resolved into more definite agencies ranged in reference to these paths. We have, then, not a world merely of chaotic and turbulent curdlings, but a world of life striving to range its material in accordance with the developing life of its organisms. In such a world "progress," "interests," "purposes," etc., first have their meanings. If we call an hypothetical chaotic world "dynamic," how much more truly may we apply the term to a world in which organizations of changes are directing this dynamic character or even furnishing new modes of it themselves. We have a dynamism referable partly to the extra-organic, partly to the organic.

We, as one or a set of these organisms, find the agencies about us falling constantly into perspectives. The world presents itself inevitably in guise of friend or foe (whether faithfully or unfaithfully) and we observe ourselves constantly approaching this and retreating from that, accepting this and rejecting that. This selective activity is guided by the relations of the actions of agents confronting us to the tendencies and interests of ourselves. Those phases of experience that lie ready to further and reinforce our natural tendencies we call good; those that thwart us we unhesitatingly denominate bad. In either case the relation of the given activity to ourselves and our own activities is what we understand as its meaning.

Certain dangers we must guard against here. To speak of the dynamic character of experience as divisable into organic and extra-organic does not imply any hard and fast distinction. The so-called "organic" end of our contrast may refer at times to the larger or the more unified part of an individual human self as *versus* smaller or less unified parts; it may refer to the interests bound up with the life of social classes or of social institutions as *versus* the tendencies of other classes or institutions, or of their own component parts.

It has just been indicated that the relation and reference of an extra-organic activity or agent to the activities or tendencies of the organism is the source of meaning. More broadly, in so far as a content is experienced, it is a meaningful content, friendly or unfriendly, good or bad, attractive or repellent. And this, now, is just what we

mean by "value." In a word we may say: the term "values" is correctly applied to those phases of experience which by virtue of their dynamic relations to our selective organic life range themselves into a more or less personal perspective.

In the light of the foregoing we may now offer the proposition that experiencedness = meaningfulness. This amounts to the statement that the philosophy of pure or immediate experience implies and presupposes a value-philosophy. Now to put the general standpoint of this paper into a nutshell, let me convert this proposition, and trust to the remainder of the paper to justify it indirectly by showing what light is then shed on certain problems formulated during the past year. *A value-philosophy implies and presupposes the philosophy of immediate experience.*

One of the questions proposed by Professor Sheldon last October[3] was whether the concept of value was unique, irreducible, ultimate, or could be reduced to terms of other categories. Compare this with the formulation by the executive committee last March,[4] namely, the relation of existence and value, and especially the detailed formulation of the four members, and it is obvious that we have here a fundamental problem.

The preceding pages have tried to show that value is primary in all senses of the word in any human experience, and that it is, therefore, a primary category in any construction of the world on the basis of experience. Had I time I should like to maintain at length that this human experience is the *only* possible *adequate* basis for such construction, that from it all intellectual enterprises rise and to it as their touchstone they return. All this, however, I shall have to presuppose.

One objection to my statement that values have been found absolutely primary in experience may run as follows: you find values primary indeed in naïve experience, but in the reflective experience that turns back for examination of the former you resolve it into the reciprocal attitudes of two dynamic agents, the central one organic, the other extra-organic. Now this involves the old question as to whether subsequent analysis destroys the unity and uniqueness of a given content of experience. Surely there is but one answer: such later analysis does not destroy the uniqueness, it is only an hypothetical ideal dissection of the experience on the basis of partial likenesses and differences that we fancy we trace. Furthermore, the ubiquity of the value-character is nowhere better shown than in the fact that the products of an act of analysis are themselves functioning as values in a new enterprise. Be it remembered,

[3] This JOURNAL, Vol. X., pages 587–588.

[4] *Ibid.*, Vol. X., pages 167–168.

finally, that analysis is prompted by, propelled by, guided by, and tested by, the actual experiencing of values as uniquenesses.

The distinction between "values" and "things" is, after all, really a relative distinction. Values are what we really have, the original data. Objects, as I have tried to show elsewhere,[5] are the result of a gradual precipitation out of a solution of general meaningfulness. They are certain constancies of import that the eager mind has seized upon in its purpose of organizing a fairly well-behaved world to live in. The ease with which such constancies of meaning become petrified into static wooden objects is only too well shown in our philosophical terminology. But the relativity of the distinction is again shown in its application to the content of a given moment's experience. If we speak of the value aspect of a given object *A*, we are for the time setting one phase of the whole content over against all the other phases. This one phase is most closely and obviously linked with our temporary interests, while the others are lumped together in the form of the material data. We are separating "value" from "thing" when we consider the beauty-worth of the painting, as over against its worth as a house decoration, as a point-to-point duplication of the represented face or landscape, as the object of expensive purchase, as something of which to be proud, as an object with which to please friends, as a bit of color immediately attracting the eye, as a heavy weight upon the hanger, as being a short distance from the floor, as having a greater dimension horizontally than vertically.

Those who hold to a hard and fast distinction between "values" and "things," are to be accused not only of oversight of the relativity of the distinction, but also of having fallen victims to that dead hand out of the past, the concept of substance. Though its explicit, corporeal form in deliberate thinking has long since been laid, its ghost is still abroad and manifested in unintentional implications. It is doubtless a natural tendency of the human mind to wish its ordered material to "stay put," but to look behind manifestations to see what the core or ground is in which these manifestations inhere, is a fruitless and bootless project. It reminds one of an interest in hidden spirits in contrast with an interest in antecedent determinants. This concept of substance involves a striking petrifaction of the various active manifestations of this our active world. The dynamic is represented as static. And so we find at the root of the distinction between "values" and "things," an over-emphasis upon the regularity and constancy and an under-emphasis upon the process and activity of experience.

A fundamental point in the formulation proposed by the four

[5] *Philosophical Review*, Vol. XXII., pages 521, 525, 531.

members is the place to be given consciousness in our study of values. This is not central to the questions offered by others during the year, but is plainly due to an interest in preserving continuity of discussion from year to year. As such it deserves notice. Let me recast the question asked to avoid other confusing issues. "Is consciousness necessary to the presence of values?" The answer may be put briefly: It is necessary in the same sense and degree and only in the same sense and degree as it is necessary for the presence of anything else—your "things," for instance. If we take the reflective point of view outlined above and consider the value-situation analyzable into organic agent striving for ends, vague or definite, and agents of the environment exerting pressure, favorable or unfavorable to these ends, we see at once that the real question is not as to whether consciousness is involved, but as to how much it is involved. Nobody knows just where the limitation between conscious and unconscious is to be drawn (if it is to be drawn). We may go into lower forms of animal life or into the complexities of human, social, and institutional life, and find on every hand cases in which the positive or negative attitude of the central agent toward surrounding agents is hardly to be called either conscious or unconscious. Whether the choice exhibited in the "motor-reflex" or "avoiding reaction" of Jennings's paramœcia is to be called conscious in a rudimentary sense, who shall say? Whether the attitude of the Republican party toward the new tariff and the currency law is to be considered a conscious attitude is likewise difficult of answer.

The element of awareness, then, we may assume to be not the critical nor the characteristic feature of valuation. It enters more characteristically in the moment of *c*-valuation. The attitudes and processes involved in evaluation are usually more highly conscious. The proof of the real character of a doubtful value as well as the resolution of a conflict between rival values in immediate experience is accomplished by a more careful analysis and definition of the values that require greater attention on the part of the experiencer. This movement of evaluation is, indeed, the heart of all intellectual life.

One of Professor Dewey's questions last May[6] was whether valuation (or what I have called evaluation) modifies antecedent values and creates new values. The answer is an unhesitating affirmative. It would seem like uttering a platitude to call attention to the fact that all the work of education in its manifold branches is directed largely by this efficient interest in modification of old values and in direct production of new. To return to the point of view of reflective analysis of human experience, this modification and creation of

[6] This JOURNAL, Vol. X., pages 268–269.

values involves modification on both sides of the value-situation, both organic interests and extra-organic environment. Education taken in its narrower sense as training the individual student to use his powers and to develop them, emphasizes the former; education taken in a broader sense as including the constant tendency to investigation and experiment and the constant work of remolding great moral and social ideals, emphasizes the latter; and yet the distinction is truly a matter of emphasis.

Here now we have a hint as to the essential nature of the distinction of subjective-objective. If, in the interests of evaluation, of reconstruction of our ideals and ends, we abstract for closer scrutiny those elements of the original value-experience that are most unmistakably connected with the organism's share—that is, with feelings, or desires, or judgments, or dispositions, or presuppositions, or the like—we are giving a statement of the *subjective elements* of the whole value. On the other hand, if we abstract those elements distinctly referable to the extra-organic part—that is, economic commodities, gods, the Altman collection, strawberries in March, posterity, the greatest good of the greatest number, etc.,—we are giving a statement of the *objective elements* of the whole value. Be it remembered that we are not making the *whole real value* either subjective or objective: we are only analyzing it and abstracting different elements for closer scrutiny, and these elements are not and can not be called the value. To have the value as a true value we must have it again in all its unanalyzed dynamic simplicity and immediacy.

It should be remarked that another use of "subjective" is certainly legitimate, namely, its application to the great class of facts found useless, hindering, confusing, and hence not entitled to pews in the congregation of the real. But this use, by emphasizing the personal equation, is really a twin brother of that just given.

In conclusion, then, we may say that a value is neither wholly organic nor extra-organic, but is an experience later analyzable into the two and their interrelation; and that however analyzable, a value still remains a *fact* as unique and primary and important as ever.

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REVIEWS AND ABSTRACTS OF LITERATURE

Principia Mathematica. A. N. WHITEHEAD AND B. RUSSELL. Volume II. Cambridge: University Press. 1912. Pp. xviii + 772.

The second volume of "Principia Mathematica" develops the three subjects of Cardinal Arithmetic, Relation Arithmetic, and Series. The last topic is not concluded. Development of Cardinal Arithmetic is based upon the "Prolegomena" which occupies the last half of Volume I.

Volume II. opens with a prefatory statement of symbolic conventions incident to the "theory of types," after which the authors take up the definition and properties of cardinal numbers. This initial topic is comparatively simple and offers a good example of the manner in which mathematical concepts are derived from simpler logical concepts,—the purpose of "Principia Mathematica."

A cardinal number (say, 5) is to be defined as the class of all classes which are similar to a given class (of five members). To this end similarity has already been defined in terms of one-one relations. Two classes, α and β, are similar when there exists at least one correlating relation which establishes a one-one correspondence of all members of α with all members of β. $[\alpha \operatorname{sm} \beta . \equiv . (\exists R) . R \in 1 \rightarrow 1 . \alpha = \mathrm{D}`R . \beta = \mathrm{C}`R \quad (*73 \cdot 1, \text{ vol. I.})]$ All classes similar to a given class of five members will be classes of five, and the class of such classes is the cardinal 5. The cardinal numbers are, then, all those classes of classes which have the relation of similarity.

The oddity of this definition lies in its "extensional" character. The cardinal number of a given class is ordinarily thought of as a *property* of the class, but the attempt so to define cardinal number would rock the "Principia" to its foundations. Throughout the work, the procedure is to determine such properties *in extension,* by logically exhibiting the class of all entities *having* the property. Hence if α be a class of five members, the cardinal number of α is the class of entities having the property "fiveness,"—*i. e.,* the class of all classes similar to α.

0 is the class of classes which are similar to the empty or null class. 1 is the class of classes similar to the class whose only member is x. This last proposition seems to be circular, but the circularity is apparent only and due to "translation" of the symbolism. More accurately, 1 is the class of classes which are similar to the class of those entities which are identical with x,—"identity" having been previously defined without the use of the idea "single" or "only" or "one." The advantages of the symbolism are well exemplified by the fact that the above complicated proposition is simply and accurately expressed as $1 = \mathrm{Nc}`\iota`x \quad (*101 \cdot 2)$. 2 is the class of all classes which are similar to some class α which has a member x and a remainder (α which is not x) whose cardinal number is 1. $[2 = \hat{\alpha}\{(\exists x) . x \in \alpha . \alpha - \iota`x \in 1\} \quad (*101 \cdot 301).]$ Although the authors do not go into the matter, it is obvious that 3, 4, . . ., might be defined by the same method. A class made up of a member, x, and a remainder whose cardinal number was 2 would have the cardinal number 3, and so on.

The further development of cardinal number requires the theory of types. This theory can not be made clear in brief space,—almost one is persuaded it can not be made clear in any space—but something of what it accomplishes may be explained. The usual discussion of the theory of number makes no question of the existence of cardinals in general. But the extensional method of definition here leads to an interesting consequence. Suppose the number of individual things were anything short of infinite,—say 7, for convenience. If the countable things be individuals, then all classes of eight will be empty classes, and the cardinal 8 will not differ from 0. The theory of cardinals beyond 7 would thus be somewhat

monotonous. Now the "Principia" does not assume an infinity of individuals; in fact, it does not assume the existence of more than one individual. If the class of individuals has one member, the class of classes of individuals has two members,—the class of one and the empty class. By such a method, the existence of classes of any (given) number of members—and the corresponding cardinals—can be proved for some sufficiently high type of entities,—classes of classes, or classes of classes of classes, or . . . . Thus, by its logical rigor, the theory of types frees mathematics from dependence on empirical data and makes arithmetic intelligible to any rational mind possessed originally of one idea. The theory of types also makes explicit the conditions of significance, or range of meaning, of various propositions, and avoids such contradictions as those discussed in the Introduction to Volume I.

The application of the theory of types leads to "homogeneous cardinals," which are never null, and in terms of these the arithmetical operations can be defined. The idea of a cardinal sum is derived from the logical concept, either . . . or. The logical sum of two classes, α and β, is the class of those things which are members of α or members of β (or members of both),—"either α or β." But in order that the logical sum should represent an arithmetical sum, it is necessary that the classes should be mutually exclusive. To secure this the authors have recourse to a device which is worth illustrating. Suppose we confine our attention to four balls, three of which are red. The cardinal number of the class "red balls" is 3; the cardinal number of "round balls" is 4. The logical sum, "balls that are either red or round," also has the cardinal number 4. But the arithmetical sum of the cardinal number of "red balls" (3) and the cardinal number of "round balls" (4) must be 7, regardless of the fact that there are only four balls in all. To this end we may substitute for "red balls" the class of couples, one member of which is a red ball and the other an *imaginary,* or non-existent, round ball. The number of such couples is the number of red balls. For the class "round balls" substitute the class of couples, one member of which is a round ball and the other an imaginary red ball. If α be the class of couples "red ball with imaginary round ball" and β the class of couples "round ball with imaginary red ball," α and β are always mutually exclusive, whether the red balls are also counted as round balls or not. Thus the *arithmetical* sum of "red balls" and "round balls" is the *logical* sum of α and β; and the concept "$+$" is not a new primitive idea, but is defined in terms of "logical constants." If m be the cardinal number of some class α, and n be the cardinal number of some class β, the cardinal $m+n$ is the class of all classes similar to the arithmetical sum of α and β.

The product $\alpha \times \beta$ is defined as the logical sum (aggregate) of couples which can be formed by taking one member of α with one member of β,—the number of combinations of members of α with members of β. If m be the cardinal number of α and n be the cardinal number of β, the cardinal $m \times n$ is defined as the class of all classes similar to $\alpha \times \beta$.

Exponentiation offers certain logical difficulties. If "m exponent n" were defined in the simplest way as the product of n factors each equal to

m, propositions about "m exponent n" would constantly require the "multiplicative axiom,"—that no product is null unless one of its factors is null. This can not be proved for infinite classes. This axiom is not assumed in general, but is inserted as an hypothesis of propositions, proof of which requires it. In order to minimize its use, the authors adopt a different definition of "m exponent n." To this end, the consideration of a new kind of product, "Prod 'κ," is necessary. "Prod 'κ" represents the possible selections from the class of classes κ. A selected class, or selection, of κ, has been previously defined (*80, Vol. I.) as a class formed by taking one "representative" from each of the constituent classes in κ. Thus if κ is made up of α (with 2 members), β (with 3), and γ (with 4), the possible selections from κ will be 24, the products of its members. Products so defined are not restricted to finite cardinals. In terms of this kind of product, "m exponent n" is defined as the product of the class of classes-of-couples which can be formed by taking members of m with a member of n. There are as many such classes-of-couples as there are members of n, and each such class has m members.

The next section (*113) is devoted to "Greater and Less," after which comes the discussion of "Finite and Infinite." The finite cardinals are treated both as "inductive" and as "non-reflexive." Inductive cardinals are those which can be reached by successive additions of 1. A reflexive class (Cantor's infinite) is similar to a proper part of itself. Inductive and non-reflexive, non-inductive and reflexive, have the same properties in general, only if the multiplicative axiom and the "axiom of infinity"—that all the finite cardinals exist—are both assumed. [The theory of types suffices to prove that any (given) finite cardinal exists, but not that *all* exist].

Inductive cardinals are defined by means of the "ancestral relation," due to Frege (See *90, Vol. I.). The ancestral relation R^* is related to the relation R as "ancestor of" is related to "parent of." If R is the relation of the cardinal n to cardinal $n+1$, the class to which 0 has the relation R^* will be the inductive cardinals. Through the definition of the ancestral relation, the use of mathematical induction is rendered wholly deductive,—though step-by-step definitions and step-by-step proofs are still necessary in some connections.

To avoid the axiom of infinity, $\aleph_0$, the smallest of the reflexive cardinals (Cantor's transfinite cardinals), is not defined as the cardinal of the class of all the inductive cardinals, but as the class of all classes which can be arranged in a progression.

In concluding the subject of cardinal arithmetic, the authors recapitulate certain conventions and the results of applying the theory of types. In the light of the rigorous discussion which has preceded "we can now adopt the standpoint of ordinary arithmetic, and can for the future in arithmetical operations with cardinals ignore differences of type" (p. 293). The reader's joy in this consummation is clouded only by the fact that we now leave the subject of cardinal number and pass to relation arithmetic.

Relation arithmetic is related to cardinal arithmetic as ordinal similarity is related to similarity in general. Cardinal numbers are classes of

similar classes; relation numbers are classes of ordinally similar relations. If ordinal m is the number of a relation, cardinal m is the number of the field (terms) of that relation. The designation "ordinal number" is usually restricted to relations which are serial and well-ordered. Hence relation arithmetic is somewhat more general than ordinal arithmetic and the properties of relation numbers belong to ordinals. The value of this greater generality is something which the reviewer has not yet discovered.

The oddities of extensional definition are nowhere so well displayed as in this part of the work. We hardly think of an ordinal number—"fourth" or "fifth"—as any kind of a relation, though it is an entity determined by its relations. But a relation taken in extension is merely the classes of terms which exhibit that order or relation. Thus "fourth" or "fifth" can be determined in extension only if a fundamental segment which it limits is logically exhibited. Hence a relation number, in "Principia Mathematica," is the number of an aggregate as much as is a cardinal number. A relation number differs from the corresponding cardinal only by requiring the aggregate to have a determined order. Thus the authors depart from the usual conception of ordinal number in ways which they do not remark.

This method of definition also leads to difficulty when we approach the ordinal number one. The "first" (in any ordered set) can not be exhibited in extension unless it have company. The relation "first" must have more than one term. But the ordinal m has cardinal m terms in its field,—ordinal 1, one term. Hence, for the "Principia," there is no ordinal number 1. The nearest approach is the *relation* number of the couple "x with x." But this relation acts like a dyad when added ordinally to another relation. Hence the addition of unity to a relation number is not the addition of the relational 1 and has to be separately treated. Finally the authors *define* the relational sum $1+1$ as 2, and frankly state that they do so in order to avoid troublesome exceptions (p. 482). After the logical niceties of the theory of types, this procedure is something of a joke.

On the whole, the treatment of relation arithmetic is a miracle of patience and ingenuity. In spite of greater complexity, the analogy of operations to those of cardinal arithmetic is preserved by clever devices of symbolism. Problems of order, in products and powers, are solved by the "principle of first differences" (due to Hausdorff), the elaboration of which requires much space.

Part V. is devoted to the treatment of serial relations, which are defined as relations of non-identity which are transitive and "connected." A relation P is connected in case, if any two members of its field, x and y, be taken, either xPy or yPx. The development of series contains less that is novel than preceding sections, and discussion of it may well be deferred to the review of Volume III., since that volume is wholly taken up in completing the subject of series.

Throughout the present volume, the authors make use of previous work in the same field. Without such studies as those of Dedekind, Cantor, Frege, and Peano, their development of the subject must have been

enormously more difficult, if not impossible. But the "Principia" has a logical rigor not previously attained, and the authors can hardly be said to have "followed" earlier treatises save in the most general way.

Although there is much that is novel in result, still the chief value of the work is its method. Proofs are ordinarily abbreviated or merely indicated, in Volume II., but they can readily be reconstructed, and when so reconstructed are identical in character with those of the "mathematical logic" of Part I. The ordinary "operations of thought" are banished. Mathematical operations are reduced to relations. The only operations used in producing proof are substitution and assertion. Expressions which are equivalent, by definition or proof, are substituted for one another, and values of variables (or expressions which may be regarded as values) are substituted for variables. If by such substitution (in a proposition already assumed or proved) there results an expression of the form "a, b, c implies x, y, z" and "a, b, c" is a previous proposition, "x, y, z" is now asserted as a new lemma or theorem. Adherence to this method is the demonstration of the assertion, first made by Mr. Russell in "Principles of Mathematics," that mathematics may be developed strictly from the fundamental logical relations.

In attempting any thorough study of the volume or of any portion, the reader will do well to familiarize himself with the operation of this method and, by reading the summaries which precede each section, to get the exact meaning of new symbols as they are introduced. He may then turn to more detailed study with profit.

The book is bound to be difficult. Explanatory passages are so condensed that they require more concentrated attention than the portion in symbols. Often one turns from the English to the symbols in order to find out exactly what is meant. Those who are inclined to object to the symbolic method of proof should defer criticism until they are able to read the symbols with understanding.

Whatever one's opinion of logistic or of the particular treatment here given to mathematics, one must at least pay his respects to the logical rigor of the method and the splendid persistence with which it is maintained. The "Principia" is to intellect what the pyramids are to manual labor. And the "Principia" has the added wonder that the whole structure is balanced on its apex of logical constants.

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JOURNALS AND NEW BOOKS.

REVUE PHILOSOPHIQUE. March, 1914. *Hasard et Déterminisme* (pp. 225–265): M. DARBON. – Chance reigns in the part of physical nature which is deprived of all teleological organization; it affects also those conscious beings who do not know how to coordinate their movements in order to attain an end or are hesitating in the choice of ends to pursue: "in both cases, chance appears as a lack of finality" . . . lack of finality meaning a lacuna in the determinism of phenomena. *Valeurs d'Art*

(*L'Esthétique sociologique*) (pp. 266–282): LUCIEN ARRÉAT. – "Society, in whatever mass, proclaims value, assesses it, but does not create value." General approval does not determine art-values, but only discloses the qualities which are necessary to value. "Sociological value and art-value are expressions which do not necessarily cover the same field." *Études expérimentales. L'Image Mentale d'après les Expériences d'Introspection:* A. SPAIER. *L'Étude de l'Image d'après les Travaux de Pavlov:* MARCELLE DONTCHEF-DEZEUZE. *Analyses et Comptes Rendus.* N. Porsenna et Serge Manolesco, *Interdépendance des Facteurs Sociaux:* J. BOURJADE. G. H. Luquet, *Essai d'une Logique Systématique et Simplifiée:* A. LALANDE. *Notices Bibliographiques. Revue des Périodiques.*

MIND. April, 1914. *Are Meanings Inherited?* (pp. 169–179): C. LLOYD MORGAN. – A review and criticism of Mr. Stout's chapter on Instinct in the new edition of his "Manual of Psychology," containing a brief interpretation of instinctive behavior on the assumption that meanings are not inherited, thus opposing the contrary view of Mr. Stout. *Psychic Function and Psychic Structure* (pp. 180–193): HENRY RUTGERS MARSHALL. – It is the current view that further advance in psychology is to be expected from functional and behavioristic rather than from structural and introspective methods. Facts are adduced to show that structural psychology is of prime importance in the solution of the fundamental problems of consciousness and of the psychical. *Some Problems of Philosophy* (pp. 194–206): F. MELIAN STAWELL. – A criticism of Mr. Russell's "Some Problems of Philosophy," questioning Mr. Russell's treatment of physical space, his Berkeleian alliance, perception and knowledge, induction, universals, hypothetical knowledge *à priori*, self-evidence, and arithmetic and the universe. *James, Bergson, and Traditional Metaphysics* (pp. 207–239): HORACE M. KALLEN. – A comparative exposition of the metaphysical views of James and Bergson, concluding that "the main outlines of Bergson's thought are the main outlines of all transcendentalism. The main outlines of James's thought are not prefigured in the history of philosophy." *Discussions: The Calculus of Strict Implication* (pp. 240–247): C. L. LEWIS. *Is Inversion a Valid Inference?* (pp. 248–250): CHARLES MERCIER. *Mr. Russell on Sense-data and Knowledge* (pp. 251–255): J. E. TURNER. *Critical Notes:* C. Mercier, *A New Logic:* E. E. C. JONES. Ch. Renouvier. *Traité de Logique Générale et de Logique Formelle:* C. D. BROAD. W. Caldwell, *Pragmatism and Idealism:* H. WILDON CARR. T. B. Muller, *De Kennisleer van het Anglo-Amerikaansch Pragmatisme:* R. F. A. HOERNLÉ. *New Books, Philosophical Periodicals. Notes and Correspondence.*

Dontchef-Dezeuze, Marcelle. L'Image et les Reflexes Conditionnels dans les Travaux de Pavlov. Paris: Felix Alcan. 1914. Pp. xvi + 176. 2 F. 50.

Griffin, E. H.; Dunlap, K.; Lovejoy, A. O. Three Studies in Current Philosophical Questions. The Johns Hopkins University Circular, No. 3. Baltimore: University Press. 1914. Pp. 99.

Vorländer, Karl. Immanuel Kant zum ewigen Frieden. Leipzig: Verlag von Felix Meiner. 1914. Pp. lvi + 74. 2.80M.

Wallas, Graham. The Great Society. New York: The Macmillan Company. 1914. Pp. xii + 383. $2.00.

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NOTES AND NEWS

At the meeting of the Aristotelian Society on July 13, Mr. H. Wildon Carr read a paper on "The Principle of Relativity and its Importance for Philosophy." The principle of relativity has been formulated in physics to account for the negative results of all experiments contrived to detect the acceleration of a movement from observations made within the moving system. It affirms that it is impossible to discover the motion of a system relatively to other systems by means of experiments performed entirely within the system (for instance, the motion of the earth relatively to sun or stars by means of purely terrestrial experiments), and that the velocity of light is a universal constant, independent of the motion of the source. The consequences of this theory are the abolition of ether, the relativity of space and time to the observer's system of reference, the impossibility of conceiving absolute position or absolute simultaneity, and that mass is a function of velocity. There were three problems of philosophy that seemed to be closely bound up with the physical problems raised by relativity. These were (1) the problem of continuity, (2) the nature of real duration, and (3) the problem of original movement. The doctrine that movement or change is original, and things are a derivation from it or views of it, was curiously in accordance with the principle that mass is a function of velocity. A discussion followed the paper, opened by Professor T. P. Nunn, who thought that Dr. Carr had over-emphasized the importance of the principle of relativity for philosophy. It represented a great mathematical advance, threw light on things badly illuminated before, but had not destroyed old views. It had, he agreed, influenced Mr. Russell, but it had not altered his old view of space so much as to make him recognize the equal reality of private space. By private space was not meant psychological space, but the real space to which each individual has access at each of his moments. Mr. Sheldon also thought the importance of the principle of relativity greatly exaggerated, and held that the experiments were explicable in many other ways. All of these were purely scientific, and had no more relation to philosophy than problems of metageometry. Dr. Wolf developed the view that the whole argument about relativity had originated in Kant's conception of time and space as modes of apprehension. He thought that the paradoxes were really due to a confusion of two different things, namely, the nature of time and space, and the difficulties of measuring time and space. Miss Constance Jones and Miss Oakeley were among the other speakers.—*Athenæum.*

We have received the fourth volume of "Die Philosophie der Gegenwart" containing an index and analysis of the philosophical literature for 1912. The work is marked by the same painstaking care, industry, and competence which have been characteristic of the previous volumes.

Dr. Harold Chapman Brown, instructor in philosophy at Columbia University, has accepted an assistant professorship in philosophy at Leland Stanford Junior University.